

----- DISCLAIMER -----

General Information:

This midterm is a *sample* midterm. This means:

- The sample midterm contains problems that are of similar, but not identical, difficulty to those that will be asked on the actual midterm.
- The format of this exam will be similar, but not identical to the actual midterm. Note that this may be a departure from the format used on exams in previous semesters!
- The sample midterm is of similar length to the actual exam.
- Note that there are concepts covered this semester that do *NOT* appear on this midterm. This does not mean that these concepts will not appear on the actual exam! Remember, this midterm is only a sample of what *could* be asked, not what *will* be asked!

How to take the sample exam:

The sample midterm should be treated like the actual exam. This means:

- “Practice like you play.” Schedule 55 minutes to take the sample exam and write answers as you would on the real exam; include appropriate justification, calculation, and notation!
- Do not refer to your books, notes, worksheets, or any other resources.
- You should not need (and thus, should not use) a calculator or other technology to help answer any questions that follow.
- The problems on this exam are mostly based on the Worksheets posted on the Math 1152 website and your previous quizzes.

However, in your future professions, you will need to use mathematics to solve many different types of problems. As such, part of the goal of this course is:

- to develop your ability to understand the broader mathematical concepts (not to encourage you just to memorize formulas and procedures!)
- to apply mathematical tools in unfamiliar situations (Indeed, tools are only useful if you know when to use them!)

There have been questions in your online homework and take-home quizzes with this intent, and there could be a problem on the exam that requires you to apply the material in an unfamiliar setting. To aid in preparation, there is such a problem on this sample exam.

How to use the solutions:

- Work each of the problems on this exam *before* you look at the solutions!
- *After* you have worked the exam, check your work against the solutions. If you are miss a type of question on this midterm, practice other types of problems like it on the worksheets!
- If there is a step in the solutions that you cannot understand, please talk to your TA or lecturer!

Math 1152

Name: _____

Sample Midterm 3

OSU Username (name.nn): _____

Autumn 2016

Lecturer: _____

Recitation Instructor: _____

Form B

Recitation Time: _____

Instructions

- You have **55 minutes** to complete this exam. It consists of 6 problems on 10 pages including this cover sheet. Page 11 has possibly helpful formulas and may also be used for extra workspace.
- If you wish to have any work on the extra workspace pages considered for credit, indicate in the problem that there is additional work on the extra workspace pages and **clearly label** to which problem the work belongs on the extra pages.
- The value for each question is both listed below and indicated in each problem.
- Please **write clearly** and make sure to **justify your answers** and **show all work!** Correct answers with no supporting work may receive no credit.
- You may not use any books or notes during this exam
- Calculators are NOT permitted. In addition, neither PDAs, laptops, nor cell phones are permitted.
- Make sure to read each question carefully.
- Please **CIRCLE** your final answers in each problem.
- A random sample of graded exams will be copied prior to being returned.

Problem	Point Value	Score
1	20	
2	12	
3	20	
4	30	
5	18	
Total	100	

1. **Multiple Choice** [20 pts]

Circle the response that best answers each question. Each question is worth 4 points. There is no penalty for guessing and no partial credit.

I. Which of the following functions is a general solution to the differential equation:

$$\frac{dy}{dt} + 2y = 2?$$

A. $y(t) = e^{-2t} + C$

B. $y(t) = Ce^{-2t} + 1$

C. $y(t) = t^2 + C$

D. $y(t) = C \cos(2t) + 2$

E. $y(t) = C \sin(2t) + 2$

F. None of the above

II. Let $f(x)$ be a continuously differentiable function whose power series is:

$$f(x) = \sum_{k=1}^{\infty} a_k (x - 4)^k.$$

Let R be the radius of convergence for this series, and suppose it is known that the series $\sum_{k=1}^{\infty} a_k$ converges. Then it MUST be true that:

A. $R \leq 1$

B. $R = 1$

C. $R \geq 1$

D. $R = \infty$; i.e. the series converges for all x .

E. None of these.

III. A point in the xy -plane is described by the *polar coordinates* $(r, \theta) = \left(1, \frac{\pi}{2}\right)$.

Which of the following gives an alternate description of the point in polar coordinates?

- A. $\left(-1, \frac{\pi}{2}\right)$ B. $\left(1, -\frac{\pi}{2}\right)$ C. $\left(-1, -\frac{\pi}{2}\right)$ D. None of the above

IV. What is the radius of convergence for the Taylor series centered at $x = 0$ for $f(x) = \frac{1}{1 - 4x}$?

- A. 0 B. 1/4 C. 1 D. 4 E. None of the above.

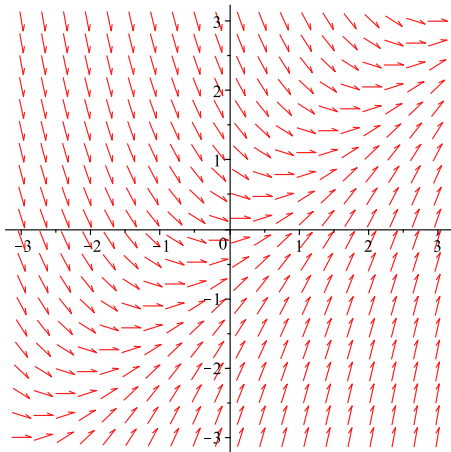
V. Which of the following is the Taylor series centered at $x = 0$ for $f(x) = x \cos(x)$?

- A. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$ B. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$ C. $\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k+1)!}$
- D. $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k)!}$ E. $\sum_{k=0}^{\infty} x^{2k+1}$ F. None of the above.

2. [12 pts] (Short Answer)

- I. Explain whether the following image could be the direction field for the differential equation:

$$\frac{dy}{dx} = y - x.$$



- II. The curve C is defined via the polar equation $r = 4 \sec \theta$. Write a description of this curve in the Cartesian coordinates x and y .

- III. Suppose that $f(x)$ is an infinitely differentiable function and it is known that $f(x)$ has a relative maximum at $x = 3$. Explain why the Taylor Series for $f(x)$ centered at $x = 3$ must be of the form:

$$a_0 + a_2(x - 3)^2 + a_3(x - 3)^3 + \dots$$

i.e. why the coefficient $a_1 = 0$. What must be the value of the relative maximum?

3. [20 pts] A curve is described parametrically by:

$$\begin{cases} x(t) &= t^3 - 3t \\ y(t) &= 1 - \frac{12}{t} \end{cases}$$

for all t where $x(t)$ and $y(t)$ are well-defined.

- I. Find $\frac{dy}{dx}$ in terms of t . Simplify your final answer!
- II. Give the Cartesian equation(s) for any vertical tangent lines to the curve or state that there are none.
- III. Give the Cartesian equation(s) for any horizontal tangent lines to the curve or state that there are none.

IV. Find the Cartesian equation for the tangent line to the curve when $t = 2$.

V. Find the Cartesian equations of any horizontal asymptotes to the curve or state that there are none.

4. [30 pts] (Taylor Series)

I. Given the power series for a function $f(x)$ is given by:

$$f(x) = \sum_{k=0}^{\infty} \frac{(x+2)^{2k}}{(2k+1)4^k}$$

A. State the center of the series.

B. Find $f(-2)$.

C. Find the radius of convergence for the power series for $f(x)$.

D. Let $g(x) = f'(x)$. What is the radius of convergence for the Taylor series for $g(x)$? Justify your response!

E. Does the series for $g(4)$ converge or diverge? Explain your answer!

F. Find $g'(-2)$.

G. Find $g^{(19)}(-2)$.

II. Given that the Taylor series centered at $x = 0$ for $\ln(1 + x)$ is given by:

$$\ln(1 + x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$$

and has radius of convergence 1:

A. Find the interval of convergence for the Taylor series for $\ln(1 + x)$.

B. Write out the Taylor series centered at $x = 0$ in summation notation for the function:

$$f(x) = \int_0^x t^2 \ln(1 + t) dt.$$

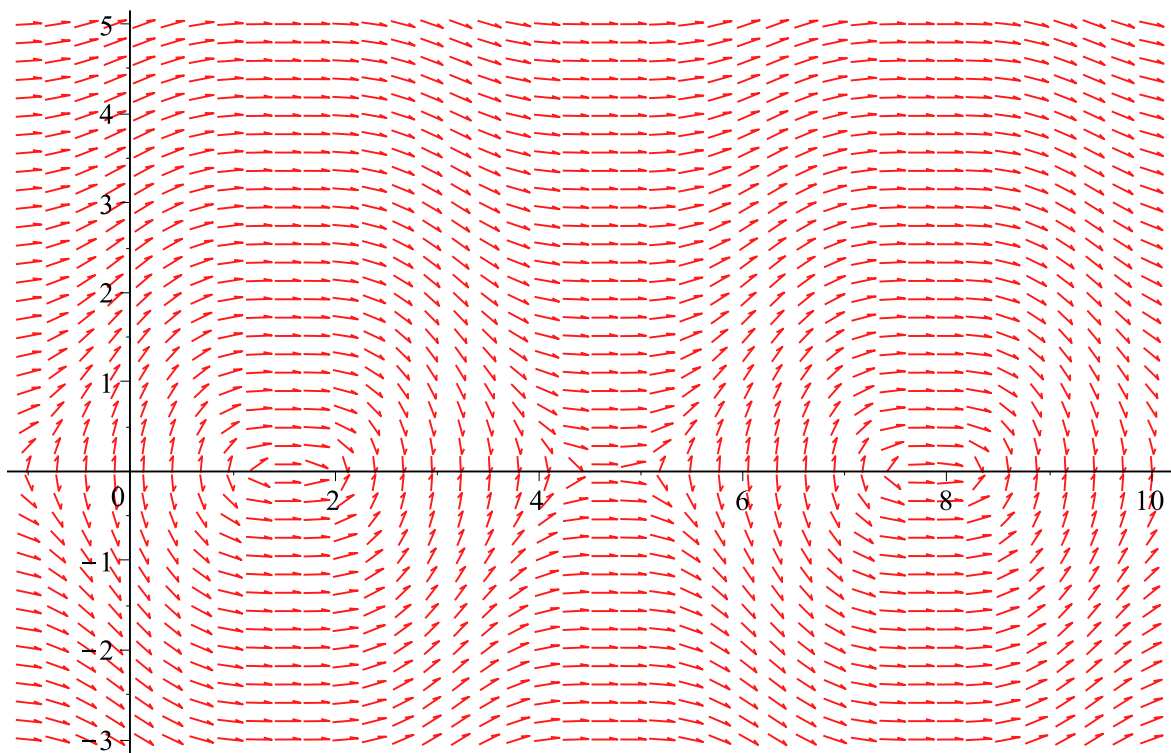
What is the interval of convergence for this series?

5. [22 pts] Consider the differential equation $\frac{dy}{dt} = \frac{3 \cos^3 t}{2y}$.

I. [1 pts] Give the order of the equation and state whether it is linear or nonlinear.

II. [3 pts] Let $y_1(t)$ be the solution to the differential equation with $y(0) = -\frac{1}{2}$ and $y_2(t)$ be the solution to the differential equation with $y(0) = 3$.

A. On the direction field below, sketch the graphs of $y_1(t)$ and $y_2(t)$ for $t \geq 0$.



B. From your direction field, does there appear to be a maximum time for which the solutions are defined? Let t_1 be the largest time for which $y_1(t)$ is defined and t_2 be the largest time for which $y_2(t)$ is defined. Using the direction field, estimate t_1 and t_2 or state that the solutions appear to be defined for all t .

III. [9 pts] Find the **general solution** to the differential equation. To get full credit, you must solve for y explicitly!

IV. [4 pts] Find the **specific solutions** $y_1(t)$ and $y_2(t)$ (defined on the previous page).

V. [5 pts] From your solution, argue why $t_1 < \frac{3\pi}{2}$ by showing that $y(t)$ is not defined when $t = \frac{3\pi}{2}$ and why $y_2(t)$ is defined for all t . Compare with the curves you drew in B.

Answers:

1. Multiple choice

- I. B.
- II. C.
- III. C.
- IV. B.
- V. F.

2. Short Answer

- I. Cannot be a solution; see solutions for an explanation.
 - II. $x = 4$
 - III. See solutions
- 3.
- I. $\frac{dy}{dx} = \frac{4}{t^4 - t^2}$
 - II. Vertical Tangent Lines: $x = -2, x = 2$
 - III. No horizontal tangent lines
 - IV. $y = \frac{1}{3}x - \frac{17}{3}$
 - V. Horizontal asymptotes are $y = 1$ as $x \rightarrow \pm\infty$.
- 4.
- I. A. Center is at $x = -2$.
 - B. $f(-2) = 1$
 - C. Radius of convergence is $R = 2$.
 - D. Radius of convergence is $R = 2$.
 - E. Diverges; see solutions for an explanation.
 - E. $g'(-2) = \frac{1}{6}$.
 - F. $g^{(19)}(x) = f^{(20)}(-2) = \frac{20!}{21 \cdot 4^{10}}$
 - II. A. IOC: $(-1, 1]$ or $-1 < x \leq 1$
 - B. Taylor series: $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2 + 3k} x^{k+3}$, IOC: $[-1, 1]$ or $-1 \leq x \leq 1$
- 5.
- I. First order, nonlinear
 - II. See solutions
 - III. $y = \pm\sqrt{3\sin t - \sin^3 t + C}$
 - IV. $y_1(t) = -\sqrt{3\sin t - \sin^3 t + 1/4}$, $y_2(t) = +\sqrt{3\sin t - \sin^3 t + 9}$
 - V. See solutions